

Quantum Evolution of the Dilaton-Inflaton Model

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ABSTRACT: The quantum evolution of a model of the universe with account of two scalar fields (*dilaton* and *inflaton*) is considered. For this case, the closed and flat models has been examined. It is shown that in both cases the realization of conditions necessary for inflation is strongly depends on coupling constant for dilaton and inflaton fields.

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1. Introduction

In Ref. [1] the new inflationary scenario (so-called extended inflation) based on a combination of old inflation [2] with Jordan-Brans-Dicke gravitational theory [3] (hereafter JBD) has been offered. It allows to slow down a speed of expansion of the universe at the initial stage of its evolution. Thus there was power-law expansion of the universe (instead of exponential one). Such delay has allowed to solve a number of problems of the old inflation noted still in Refs. [1] and [4] (in particular, the problem of graceful exit). Unfortunately in model of La and Steinhardt there was the basic difficulty connected with impossibility of an establishment of thermal equilibrium in walls of bubbles for cosmologically reasonable times. It results in violation of isotropy of background radiation. With the purpose of avoidance of this problem, it is necessary that the coupling constant in JBD theory has been limited from above $\omega \leq 20$ [5]. However, the lower observable bound is $\omega \geq 500$ [6]. For resolving of this problem, Damour, Gibbons and Gundlach [7] (hereafter DGG) has proposed the model in which, except of visible matter, the invisible (field) component is present as well. Action in their model has a form (in Jordan (physical) frame):

$$S[g_{\mu\nu}, \Phi, \psi_V, \psi_I] = S_{BD}[g_{\mu\nu}, \Phi] + S_V[g_{\mu\nu}, \psi_V] + S_I[g_{\mu\nu}, \Phi, \psi_I], \quad (1.1)$$

where Φ is JBD scalar field (*dilaton*), $S_{BD}[g_{\mu\nu}, \Phi]$ - Brans-Dicke action [3]

$$S_{BD}[g_{\mu\nu}, \Phi] = \int d^4x \sqrt{-g} \left[-\Phi R + \frac{\omega}{\Phi} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi \right], \quad (1.2)$$

and S_V , S_I are actions for visible ψ_V and invisible ψ_I components of material fields. As the usual matter is described as an ideal fluid, it does not play any role during inflation at

this stage of evolution of the universe and it is possible to dropped it. Invisible component in DGG model reads:

$$S_I[g_{\mu\nu}, \Phi, \psi_I] = \int d^4x \sqrt{-g} \left[\Phi^{1-\beta} g^{\mu\nu} \partial_\mu \psi_I \partial_\nu \psi_I - \Phi^{2(1-\beta)} V(\psi_I) \right], \quad (1.3)$$

where $\beta \equiv \beta_I/\beta_V$ and $\beta_{V,I}$ are coupling constants of visible and invisible matter with dilaton field. (Hereafter we work in geometrical units, i.e. $c = 1$ and $8\pi G_N = 1$, where G_N is the current value of Newtonian constant.) DGG worked in so-called Einstein frame which turns out from Eq. (1.1) via conformal transformation of the metrics $\tilde{g}_{\mu\nu} = 2\Phi g_{\mu\nu}$ with simultaneous redefinition of the variable $\Phi = \frac{1}{2} \exp(\phi/\phi_0)$, where $\phi_0 \equiv \sqrt{\omega + 3/2}$. Using Eq. (1.1) Holman, Kolb and Wang [8] has shown that at a choice of inflaton field as an invisible matter, it is possible to find region on a plane $\omega - \beta$ in which the mentioned above restrictions $\omega \leq 20$ (necessary for successful inflation) and $\omega \geq 500$ (claimed from observations) can be satisfied simultaneously. It is possible because, instead of standard restriction $\omega \leq 20$ from model [1], the new relation $\omega/\beta^2 \leq 20$ will work. It can be easily satisfied at $\omega \geq 500$.

The further development of the model was reduced to introduction in Eq. (1.3) arbitrary exponents n and m instead of $(1 - \beta)$ and $2(1 - \beta)$ [9] that has allowed to investigate the generalized process of nucleation and percolation and to calculate time-depended nucleation rate per unit volume. In Ref. [10] the restrictions on parameters n and m at which the sufficient inflation was provided have been found.

It seems interesting to consider quantum evolution for the model of extended inflation. The main purpose of this paper is to find necessary conditions for ensuring of inflation on post-quantum stage. In Section 2 the Wheeler-DeWitt equation for closed, open and flat model is obtained in general form. Section 3 is devoted to problem statement and to general description of solution technique. The derivation and solving of corresponding equations for closed and flat models are presented in Section 4. Finally, in Section 5 the probabilities of tunneling and above the barrier reflection for closed and flat models accordingly are found.

2. Wheeler-DeWitt Equation

We will work in Einstein frame. Thus, using conformal transformation mentioned in Introduction and using designation $b = 1 - \beta$, we can obtain from Eqs. (1.2) and (1.3) the following Lagrangian (see also Ref. [11]):

$$\mathcal{L} = \frac{1}{2} \sqrt{-\tilde{g}} [-\tilde{R} + \tilde{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + e^{b\phi} \tilde{g}^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - e^{2b\phi} m^2 \sigma^2]. \quad (2.1)$$

Here the massive inflaton field σ with the simplest form of potential $V = \frac{1}{2} m^2 \sigma^2$ is using as invisible matter. Let us write out the general expression for metrics for closed, flat and open universes:

$$ds^2 = N^2(t) dt^2 - a^2(t) \left[d\chi^2 + \left(\frac{\sin \sqrt{k} \chi}{\sqrt{k}} \right)^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \right]. \quad (2.2)$$

In this expression $k = -1, 0, 1$ are for closed, flat and open models accordingly, $N(t)$ is lapse function. As is known, the metrics in itself does not yet defines global topology of space-time. It enables to consider spacelike sections as compact in a case of open and flat models. Thus the volume, limited by spacelike hypersurfaces, remains finite and have a form:

$$V_k = \int d^3x \sqrt{-h}. \quad (2.3)$$

(h is the three-dimensional metrics.) In a case of homogeneous model, using Eq. (2.2), we can obtain:

$$V_k = a^3 v_k, \quad v_k = \int d^3x \frac{\sin^2 \sqrt{k} \chi}{k} \sin \theta, \quad (2.4)$$

where the value of v_k depends on topology of the spacelike sections (see, e.g., Ref. [12] and references inside). Then, using Eqs. (2.1) and (2.2), we have the following Lagrange function:

$$L = 3v_k N \left[ka - \frac{1}{N^2} a \dot{a}^2 \right] + \frac{1}{2} v_k N a^3 \left[\frac{1}{N^2} \dot{\phi}^2 + \frac{e^{b\phi}}{N^2} \dot{\sigma}^2 - e^{2b\phi} m^2 \sigma^2 \right]. \quad (2.5)$$

Further, using Eq. (2.5) and calculating the conjugate momenta

$$p_a = -\frac{6v_k}{N} a \dot{a}, \quad p_\phi = \frac{v_k}{N} a^3 \dot{\phi}, \quad p_\sigma = \frac{v_k}{N} a^3 e^{b\phi} \dot{\sigma}, \quad (2.6)$$

we can obtain the corresponding canonical Hamiltonian:

$$H = \frac{N}{v_k} \left[-\frac{1}{12} \frac{p_a^2}{a} + \frac{1}{2a^3} \left(p_\phi^2 + e^{-b\phi} p_\sigma^2 \right) - 3v_k^2 a \left(k - \frac{1}{6} m^2 a^2 e^{2b\phi} \sigma^2 \right) \right]. \quad (2.7)$$

Quantization of Eq. (2.7) is reduced to replacement of the conjugate momenta p_a, p_ϕ, p_σ on operators $-i\partial/\partial a$, $-i\partial/\partial \phi$ and $-i\partial/\partial \sigma$ accordingly. Wheeler-DeWitt equation turns out at action of the Hamilton operator on the wave function $\psi(a, \phi, \sigma)$ and looks like:

$$-\frac{1}{12} a^{-p} \frac{\partial}{\partial a} \left(a^p \frac{\partial \psi}{\partial a} \right) + \frac{1}{2a^2} \left(\frac{\partial^2 \psi}{\partial \phi^2} + e^{-b\phi} \frac{\partial^2 \psi}{\partial \sigma^2} \right) + 3v_k^2 a^2 \left(k - \frac{1}{6} m^2 a^2 e^{2b\phi} \sigma^2 \right) \psi = 0, \quad (2.8)$$

where p is factor ordering which introduction is caused by ambiguity of commutation properties of the operators a and p_a . Transformation of wave function

$$\psi \rightarrow a^{-p/2} \psi$$

allows us to exclude the first derivative in Eq. (2.8) that gives:

$$-\frac{1}{12} \frac{\partial^2 \psi}{\partial a^2} + \frac{1}{2a^2} \left(\frac{\partial^2 \psi}{\partial \phi^2} + e^{-b\phi} \frac{\partial^2 \psi}{\partial \sigma^2} \right) + \left[-\frac{p}{24a^2} \left(1 - \frac{p}{2} \right) + 3v_k^2 a^2 \left(k - \frac{1}{6} m^2 a^2 e^{2b\phi} \sigma^2 \right) \right] \psi = 0. \quad (2.9)$$

Let us choose the factor ordering $p = 1$. Also we shall use the following redefinitions: $\phi, \sigma \rightarrow \sqrt{6}\phi, \sigma$, $b \rightarrow \sqrt{1/6}b$, $v_k \rightarrow 1/6v_k$. Finally, from Eq. (2.9) we have:

$$-\frac{\partial^2 \psi}{\partial a^2} + \frac{1}{a^2} \left(\frac{\partial^2 \psi}{\partial \phi^2} + e^{-b\phi} \frac{\partial^2 \psi}{\partial \sigma^2} \right) + W \psi = 0, \quad (2.10)$$

where superpotential

$$W = -\frac{1}{4a^2} + v_k^2 a^2 \left(k - a^2 e^{2b\phi} m^2 \sigma^2 \right). \quad (2.11)$$

3. Problem statement

The obtained equation describes quantum evolution of the universe. Thus the statement of problem essentially depends on type of a model of the universe which is examined: closed, flat or open. In view of equality to zero of full energy of the universe, it is clear from form of superpotential (2.11) that consideration of the problem is reduced to research either the process of tunneling in case of the closed model or the process of above the barrier reflection of the universe's wave function in case of flat and open models. Such consideration was carried out repeatedly in various statements. However, in most cases one-dimensional problems were examined. In this cases wave function depends only on one variable - scale factor. At that it was possible to obtain exact solutions in a number of problems [13]. And, that is even more important, in one-dimensional statement it is seems possible to find clear physical sense of the wave function. In a case of multidimensional problems, when the wave function depends also from field variables, the problem is complicated not only by mathematical difficulties but also by complexities with understanding of physical sense of the multidimensional wave function. In this connection some authors proposed the methods of reducing of multidimensional problems to one-dimensional one. It allows to give to the wave function physically clear sense [14].

In the given work we shall use one of such ways which was suggested in Ref. [15]. The essence of this method consists in consideration of semi-classical evolution of a model of the universe. In this case the corresponding Hamilton-Jacobi equation, similar to the equation from classical mechanics, is written down for the Wheeler-DeWitt equation. This equation is nonlinear first-order partial differential equation. For its solution the known method is used. The essence of this method consists in representation of the given non-linear partial differential equation through a system of the first-order ordinary differential equations (so-called set of characteristic equations). The given equations describe a set of characteristics which are form an integral surface of initial partial differential equation. Distinctive feature of the system of characteristic equations is that the functions, entering in equations, depend only on one argument. Thus, initially multidimensional problem is reduced to one-dimensional one that allows to find physically sensible one-dimensional wave functions along each of characteristics. Then for calculation, for example, of permeability of a barrier, it is necessary only to sum and to average permeability on all characteristics.

4. Derivation and solving of a set of characteristics

Let us pass directly to research of our model.

Closed model

At the beginning, let us consider the case of closed universe. As it was already specified above, in this case the process of tunneling of the wave function will be take place. In semi-classical treatment of the given problem we can present the wave function as $\psi = e^{-S}$. Then, being limited to zero approximation, we obtain the following equation for action S

from Eq. (2.10):

$$-\left(\frac{\partial S}{\partial a}\right)^2 + \frac{1}{a^2} \left[\left(\frac{\partial S}{\partial \phi}\right)^2 + e^{-b\phi} \left(\frac{\partial S}{\partial \sigma}\right)^2 \right] + W = 0. \quad (4.1)$$

The corresponding set of characteristics under a barrier will be (see., e.g., Ref. [16]):

$$\begin{aligned} \frac{d\phi}{da} &= -\frac{q}{a^2 F}, & \frac{d\sigma}{da} &= -\frac{e^{-b\phi} v}{a^2 F}, \\ \frac{dq}{da} &= -\frac{1}{2} \frac{be^{-b\phi} v^2}{a^2 F} - \frac{bv_k^2 a^4 e^{2b\phi} m^2 \sigma^2}{F}, & \frac{dv}{da} &= -\frac{v_k^2 a^4 e^{2b\phi} m^2 \sigma}{F}, \\ \frac{dS}{da} &= \frac{W}{F}, \end{aligned} \quad (4.2)$$

where $q = \partial S / \partial \phi$ and $v = \partial S / \partial \sigma$ are the generalized momenta and $F = \sqrt{(q^2 + v^2)/a^2 + W}$.

This equations describe a process of tunneling of a "particle" along the characteristics. Each of such characteristics differs from another by definition of corresponding initial conditions. For this purpose let us analyse the potential (2.11) at $k = 1$. As we see, the whole minisuperspace is divided into three areas (see Fig. (1)): 1) internal area, where

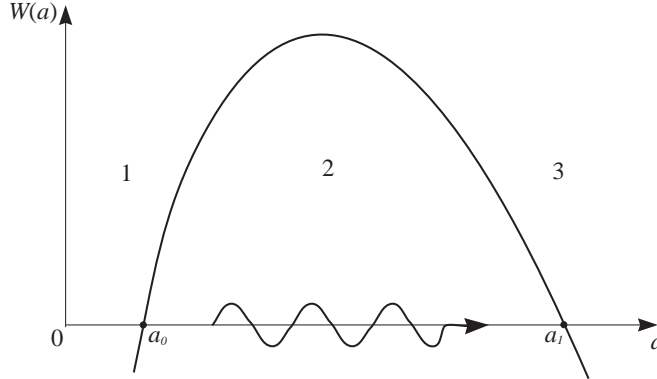


Figure 1: The conventional form of an one-dimensional barrier is shown. Tunneling of the wave function occurs from the point a_0 , where initial conditions are set, into the point of an output from under the barrier a_1 . The initial conditions in a_0 define the form of a barrier which is individual for each characteristic.

there is no classical space-time and the metrics is under the strong quantum fluctuations; 2) classically forbidden area under a barrier; 3) classically allowed area. Initial conditions are set on bound between internal and classically forbidden areas. For finding of the bound, it is necessary to equate superpotential (2.11) to zero. In generally, it will be a surface. However, if to choose as initial conditions $\phi, \sigma = 0$, evolution of the model will begin in a point $a_0 = (1/4v_1^2)^{1/4}$, where v_1 is a three-volume of the closed universe. In case of sphere $v_1 = 2\pi^2$. Further, we should set initial momenta $q = q_0$ and $v = v_0$ which will vary from 0 up to ∞ . So, all characteristics will be differ by different values of momenta. Using the specified initial conditions and solving numerically the set of equations (4.2) we have obtained the following results (see Figs. (2)).

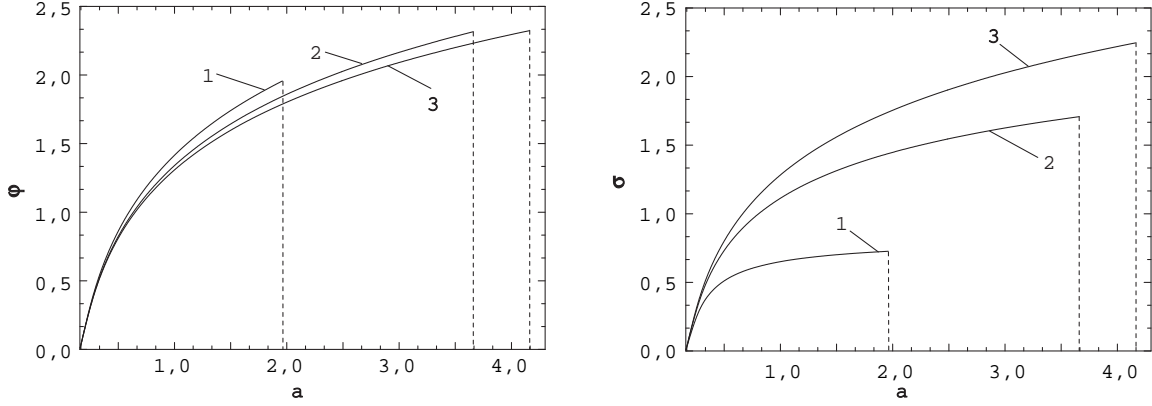


Figure 2: The dependence $\phi(a)$ (left panel) and $\sigma(a)$ (right panel) for different b : 1 - for $b = 1$, 2 - for $b = 0.2$, 3 - for $b = 0.02$. Dotted lines are pointing out the points of output from under a barrier.

This implies that in process of under a barrier evolution, there is a growth both inflaton and dilaton fields. Thus it is important to note that thickness of a barrier and, consequently, the rate of growth of fields strongly depends on parameter b : the less value of b means the more width of a barrier and the more value of fields on an output from under a barrier. On the other hand it is clear that after an output from under a barrier the parameters of the model should provide conditions for the subsequent inflationary stage. At least, there are two such conditions: 1) inflaton field should be the Planck's order; 2) it should varies slowly enough to ensure long inflationary stage. In the given model such conditions are provided in a case when the coupling constant b is small enough. This conclusion is in agreement with specified in Ref. [11] restriction on value of $b < 1$.

Flat model

It is possible also to consider evolution of the given model for cases when $k = 0, -1$. It is obvious that in these cases the potential barrier will be lay in negative area and only above the barrier reflection is possible. In view of absence of principle distinctions of process of above the barrier reflection for open and flat model, we shall consider, for example, an evolution of flat model (i.e. we should put in Eq. (2.11) $k = 0$). Since full energy of the universe is equal to zero and the superpotential always is less than zero, then in case of flat model evolution of the universe will always take place in classically allowed area. In this connection let us search for solution of equation (2.9) as $\psi = e^{-iS}$. Then we obtain:

$$-\left(\frac{\partial S}{\partial a}\right)^2 + \frac{1}{a^2} \left[\left(\frac{\partial S}{\partial \phi}\right)^2 + e^{-b\phi} \left(\frac{\partial S}{\partial \sigma}\right)^2 \right] - W = 0. \quad (4.3)$$

Then the set of characteristics reads:

$$\frac{d\phi}{da} = -\frac{q}{a^2 F}, \quad \frac{d\sigma}{da} = -\frac{e^{-b\phi} v}{a^2 F},$$

$$\begin{aligned}\frac{dq}{da} &= -\frac{1}{2} \frac{be^{-b\phi}v^2}{a^2F} + \frac{ba^4e^{2b\phi}m^2\sigma^2}{F}, & \frac{dv}{da} &= \frac{a^4e^{2b\phi}m^2\sigma}{F}, \\ \frac{dS}{da} &= -\frac{W}{F},\end{aligned}\tag{4.4}$$

where $F = \sqrt{(q^2 + v^2)/a^2 - W}$. (Here we have choose $v_0 = 1$ (see Ref. [12]).) For numerical research of this system, it is necessary to set the certain boundary conditions. It can be made as follows. Two conditions, necessary for the beginning of inflationary stage, were specified above. First of these conditions can be written as the following restriction on value of inflaton and dilaton fields in the beginning of inflation [11]:

$$e^{2b\phi}m^2\sigma^2 = 1.\tag{4.5}$$

The fulfilment of the second condition can be checked directly at numerical research of the set of equations (4.4). Thus evolution of the universe will pass in two stages: first, the stage of contraction will occurs from a point of the beginning of inflation up to a turn point. Then, after the bouncing off, the model will come on a stage of expansion.

So, having use the condition (4.5) and, as well as in a case of closed model, setting some values of momenta $q = q_0$ and $v = v_0$ in a point of the beginning of inflation we can carry out numerical research of the set of characteristics. Its results are presented in Figs. (3).

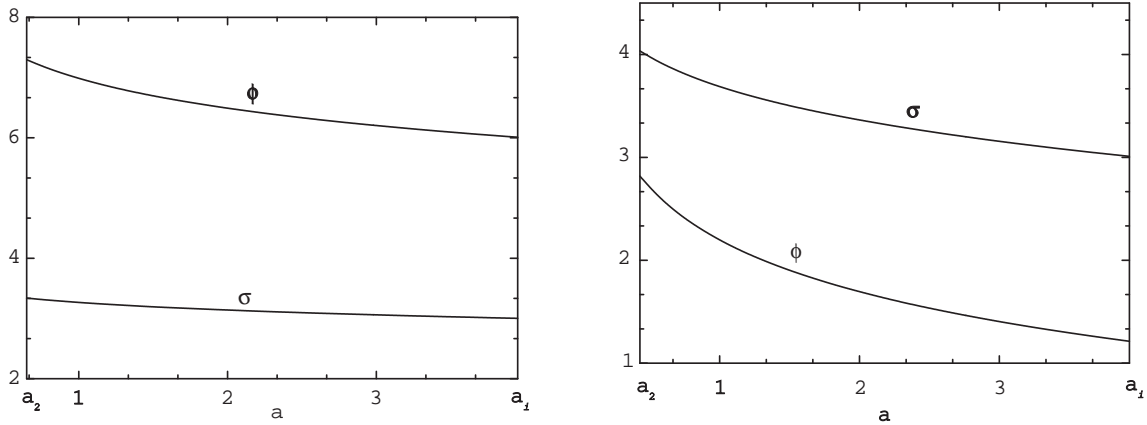


Figure 3: The dependence $\phi(a)$ and $\sigma(a)$ for $b = 0.2$ (left panel) and for $b = 1$ (right panel). Here a_2 is a point of above the barrier reflection, a_i - point of the beginning of inflation.

5. Probabilities of tunneling and above the barrier reflection of wave function

In the considered above models, evolution of scalar fields was investigated during tunneling and above the barrier reflection. However, it is also interesting to find probabilities of these processes. It appears that it is possible to make in analytical form because of one feature

of the considered sets of characteristics. It is clear from the numerical analysis of Eqs. (4.2) in case of tunneling that action S is large at small values of momenta q and v . Thus permeability of a barrier will be exponentially small in accordance with the formula

$$D = \exp(-2S). \quad (5.1)$$

Therefore the basic contribution to permeability will be brought by characteristics with large momenta for which the action is small. In this case it is possible to simplify essentially the set of characteristics (4.2) because at large momenta's values their practically does not vary during tunneling [15, 17]. Then Eqs. (4.2) will become:

$$\begin{aligned} \frac{d\phi}{da} &= -\frac{q}{a^2 F}, & \frac{d\sigma}{da} &= -\frac{e^{-b\phi} v}{a^2 F}, \\ q &= \text{const}, & v &= \text{const}, \\ \frac{dS}{da} &= \frac{W}{F}, \end{aligned} \quad (5.2)$$

where $q, v \gg 1$, $F \approx A/a$ and $A^2 = q^2 + v^2$. Then it is easy to find

$$\phi = \frac{q}{A} \ln \frac{a_0}{a}, \quad \sigma = -\frac{v}{bq} \left(\frac{a}{a_0} \right)^{bq/A} \quad (5.3)$$

and

$$S = \frac{1}{A} \left[-\frac{1}{4} \ln \frac{a_*}{a_0} + v_1^2 \frac{1}{4} (a_*^4 - a_0^4) - \frac{m^2 v^2}{b^2 q^2} v_1^2 \frac{1}{6} (a_*^6 - a_0^6) \right]. \quad (5.4)$$

In these expressions a_0 is a point of an entrance of wave function under a barrier, a_* is a point of an output. Last one is determined from the condition $W(a_*, \phi_*, \sigma_*) = 0$ and, finally, is a function of momenta q and v .

As it has already been mentioned above, these formulas are true only at large values of q and v . However, in this case action S is small that does not allow to use the usual formula for permeability of a barrier (5.1) which is applicable only at large S . Therefore in this case it is necessary to use the formula of generalized WKB [18]:

$$D = \frac{\exp(-2S)}{1 + \exp(-2S)}, \quad (5.5)$$

which is used at any S . Then, using Eqs. (5.4), (5.5) and averaging on all characteristics, we can obtain the following expression for permeability of a barrier:

$$D = \lim_{\bar{q}, \bar{v} \rightarrow \infty} \frac{1}{\bar{q} \bar{v}} \int_0^{\bar{q}} \int_0^{\bar{v}} dq dv \frac{\exp(-2S(q, v))}{1 + \exp(-2S(q, v))}. \quad (5.6)$$

In a case of flat model the above the barrier reflection will occurs near to a maximum of superpotential (2.11) where it is small on absolute value in comparison with the term $(q^2 + v^2)/a^2$ from Eqs. (4.4). Then we shall have again the same expressions (5.3) for fields. For calculation of reflectivity we use the known formula [19]:

$$R = \exp \left(-4 \operatorname{Im} \int_{a_1}^{a_2} da \sqrt{-W(a)} \right), \quad (5.7)$$

where a_1 is an arbitrary point on a real axis and a_2 is a turn point calculated from the condition $W = 0$. In our case, using Eqs. (5.3) and (2.11) for $k = 0$, we have:

$$W = -\frac{1}{4a^2} - \frac{m^2 v^2}{b^2 q^2} a^4 \quad (5.8)$$

and

$$a_2 = \left(\frac{bq}{2mv} \right)^{1/3} \exp[i\pi(2n+1)/6]. \quad (5.9)$$

Then the reflectivity, averaged on all momenta, will be:

$$R = \lim_{\bar{q}, \bar{v} \rightarrow \infty} \frac{1}{\bar{q} \bar{v}} \int_0^{\bar{q}} \int_0^{\bar{v}} dq dv \exp \left(-\frac{\pi}{3} \sqrt{\frac{m}{b} \frac{v}{q}} \right). \quad (5.10)$$

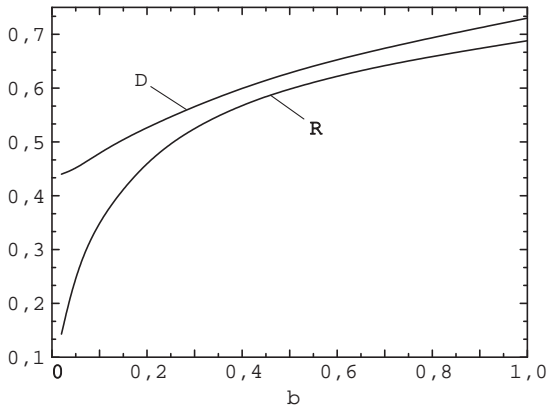


Figure 4: The dependence of permeability D and reflectivity R from b .

The obtained formulas (5.6) and (5.10) allow us to find the probabilities at various values of parameter b (Fig. 4). As we see from figure, in case of the closed model permeability is the larger when b is larger (i.e. when the barrier is more thin), that is quite natural. On the other hand, as was noted above, than b is smaller, the inflaton field on an output from under a barrier is larger and the conditions for inflation are better. So, certain compromise is necessary at a choice of parameter b for maintenance both enough long inflation and high probability of tunneling.

In a case of flat model the similar situation occurs. Here, then b is larger, decreasing of the inflaton field σ is faster on the expansion stage (see Figs. (3)). Then the conditions for inflation at identical values of the inflaton field in the point of above the barrier reflection a_2 in a case of large b will be worse than at small b . At that, the probability of above the barrier reflection is increasing at large b , as in a case of closed model.

I am grateful to H.-J. Schmidt and H. Kleinert for discussions. This work was supported by ISTC project KR-677.

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